Search Models and Kinked Demands

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Abstract

I show that consumer search leads to price indeterminacy. Provided search costs are continuously distributed across consumers and there is a positive density of consumers with zero search costs, consumer search induces continuous but kinked demand curves. The kink arises from the constitutive feature of search models: consumers' imperfect information. Because consumers cannot observe a firm's price before visiting it, a firm loses customers with a price increase, but attracts none with a price decrease. The kink in demand in turn leads to price indeterminacy: a whole interval of prices is consistent with equilibrium. The indeterminacy is different from the one recently highlighted in models of labor search: it pertains to markets where prices are publicly quoted, not bilaterally bargained, and thus applies to goods markets. Just as the bilateral monopoly indeterminacy opens the door to wage rigidity, the kinked-demand indeterminacy opens the door to price rigidity.

Introduction

The Walrasian theory of value gives an answer to how prices are determined: so that they clear markets. Starting with Stigler (1961), the search literature has stressed one ubiquitous empirical challenge to the concept of market-clearing price: in most markets, the law of one price fails in favor of price dispersion. To explain price dispersion, the search literature has put one assumption on the front scene: that *"unless a market is completely centralized, no one will know all the prices which various sellers quote at any given time"*.[1]

The search framework has challenged the idea of a market-clearing price. Yet it has not challenged a more general notion: price determinacy. In most of the search literature, even when there is no such thing as *the* equilibrium market price, there usually exists a unique equilibrium *distribution of* prices, that arises at the (broadly understood) crossing of demand and supply.

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[1] The quote is from Stigler (1961). On the various ways to get a search model to generate price dispersion in equilibrium, see e.g. McMillan and Rothschild (1994)'s survey.

I show that the constitutive feature of search models—buyers' imperfect information on prices—challenges price determinacy too. Buyers' asymmetric information on all the prices charged in a market gives rise to an asymmetry in firms' demand curves: because buyers only know the price charged at a store after they visited it, firms fail to attract many more consumers with price decreases, but they do lose many with price increases. Formally, demand curves are kinked. The kink in turn leads to price indeterminacy. The informational intuition for a kink in demand dates back to Scitovsky (1978), Stiglitz (1979) and Woglom (1982). In a recent paper—Dupraz (2016)—I provide a microfounded theory of the argument in a general-equilibrium switching-cost model with asymmetrically informed consumers, and draw its implications for price rigidity and monetary policy.

In this note, I rederive the argument in a partial-equilibrium search model. The proof for the search model is mostly the same: a search model *is* essentially a switching-cost model with asymmetrically informed consumers.[2] Yet, the purpose of this note is to clarify two points. First, why haven't earlier papers in the search literature found kinked demand curves? I spell out the two minimal assumptions required for a continuous but kinked demand curve: that the distribution of search costs among consumers be continuous, and that there exist a positive density of consumers with zero search costs. Many search models assume instead that all consumers face the same search cost. This creates a discontinuity in demand curves, and leads to Diamond (1971)'s paradox: arbitrarily small search costs prevents competition to drive prices below their monopolistic level. Simply put, the discontinuity in demand hid the more generic kink in demand, and the resulting price indeterminacy.

Second, I discuss the connection between the price indeterminacy obtained here and the one emphasized in models of labor search by Hall (2005) and Shimer (2005). They differ: the Hall-Shimer indeterminacy applies to search markets where prices are bilaterally bargained once a buyer and a seller have matched. It arises from the infinity of ways to share the surplus between the seller and the buyer in the case of a bilateral monopoly. In contrast, the kinked-demand indeterminacy applies to markets where prices are publicly quoted ex-ante, not bilaterally bargained ex-post. It thus applies to goods markets. Yet both forms of indeterminacy convey the same message: in markets with search, not only can the concept of market-clearing price be blurred into a market-clearing distribution of prices: it can entirely dissolve into price indeterminacy. Since the market-clearing price does not exist, the distinction between classical market-clearing theories of fluctuations and Keynesian non-market-clearing theories is elusive.

[2]A benefit of a switching-cost model over a search model is that it is allows to distinguish between the assumption of a cost to moving to competitors, and the assumption of consumers' asymmetric information on prices. In contrast, in a search model, the cost of moving to another firm and the cost of gathering information on prices are conflated in the search cost. In Dupraz (2016), I consider the switching-cost model under full information and show it produces no kink in demand, highlighting that the assumption driving the kink is asymmetric information, not switching costs.

1 The kinked-demand price indeterminacy

In this section, I show the two main results of this note: consumer search induces kinked demand curves, and kinked demand curves induce price indeterminacy. Consider the following partial-equilibrium model of search. A continuum of firms $k \in [0,1]$ produce a homogeneous good at the same (constant-return-to-scale) cost c. Their pricing decisions result in a distribution of prices in the market. A consumer within a continuum $[0,1]$ finds a firm to shop at by drawing from the distribution. The first draw is free, then search is sequential: after each new draw, consumer j can draw again at a search cost $\gamma \geq 0$, the same for all draws but different across consumers. An equilibrium is when the pricing decisions of firms and the shopping decisions of consumers are individually optimal. I consider symmetric equilibria where the distribution of prices is degenerated to a Dirac at a single market price p^*. (Much of the good-market search literature focused on ways to get price dispersion in equilibrium; I don't.)

1.1 Consumers' behavior

I assume all consumers have the same quasi-linear preferences over the good, and cash. The utility of a consumer is, noting d the consumption of the good and p the price he ends up paying:

$$u(d) - p.d. \tag{1}$$

The consumer's problem is to decide both where to shop and, once he has settled on a store, how many units to buy. Solving the problem backward, once he has decided on a shop, a consumer buys:

$$d(p) = (u')^{-1}(p) \tag{2}$$

units of the good. This individual demand curve d is the demand curve that a monopolist would face. I make the standard assumption that d is continuous and differentiable: there is no kink in the demand of an individual consumer.

Turn to the consumer's search decision. Note v the indirect utility of a consumer when buying at price p:

$$v(p) \equiv u(d(p)) - p.d(p). \tag{3}$$

Note γ the cost for the consumer of drawing a new firm from the distribution—the search cost. (Crucially, I will let the search cost differ across consumers, but to solve the consumer's problem, this is irrelevant.) As is intuitive—and rigorously derived in the appendix—a consumer draws a new firm when the cost of a new draw is less than the benefits of the draw in terms of increased (indirect) utility:

Lemma 1. *Let p^* be the market price charged at all firms in a symmetric equilibrium. A consumer who* *draws a firm charging a price p and faces a search cost γ draws another firm if and only if:*

$$\gamma < v(p^*) - v(p). \tag{4}$$

1.2 Demand curves

Given this behavior of consumers, consider the demand curve faced by a firm $k \in [0,1]$, taking the pricing decision p^* of all competitors as given. Firm k's demand depends on how many customers firm k attracts—the extensive margin of demand—and how much each customer buys—the intensive margin of demand. The extensive margin of demand is driven by the searching decision of customers. At the first random draw of search, firm k got a fraction dk of customers. I assume customers differ in their search costs γ, and note F (f) the CDF (pdf) of the distribution of search costs among customers—the same in all subgroups defined by each firm's initial customers. If firm k sets its price p^k at the market price, no customer leaves nor comes: k's market share remains dk. If firm k sets it price above the market price $p^k > p^*$, its customers with a low enough search cost γ—lower than the threshold defined in (4)—draw another firm and leave to their new draws. Firm k only retains its customers with a high enough search cost: $[1 - F(v(p^*) - v(p^k))]dk$ customers. If firm k sets its price below the market price $p^k < p^*$, none of its customers leaves, but no customer comes from other firms either, so firm k gets exactly dk customers. Combined with the individual demand (2) which determines the intensive margin of demand, this determines firm k's demand:

$$D^k(p^k, p^*) = \begin{cases} d(p^k)dk & \text{if } p^k \leq p^*, \\ [1 - F(v(p^*) - v(p^k))]d(p^k)dk & \text{if } p^k \geq p^*. \end{cases} \tag{5}$$

The first result of this note is that under minimal assumptions on the distribution of search costs F, this demand curve is continuous but kinked.

Assumption 1. *The distribution of search costs F is continuous (it has no mass).*

Assumption 2. *There is a positive density of consumers with no search cost: $f(0) > 0$.*

Proposition 1. *Under assumptions (1) and (2), a firm's demand is continuous everywhere, and kinked at the market price $p^k = p^*$.*

Both assumptions are mostly technical. A continuous distribution of search costs guarantees a continuous demand curve. A positive density of consumers with no search costs guarantees that the market share is upward-elastic: if the price increases, some customers leave. The reason for the kink in demand is not hidden in these two assumptions, but arises instead from the constitutive assumption of the search framework: consumers' imperfect information. Precisely, what matters is that a consumer has *asymmetric* information on the prices charged at the different firms. He perfectly observes the price at the firm he has drawn, but only observes the distribution of prices at other firms. Thus, he observes changes in prices at one store, but not at others. Therefore, a firm faces the extensive elasticity of demand only upward: if it increases its price above the market price, all of its customers notice and those with low enough search costs draw another firm; if it decreases its price below the market price, no new customer notices, so none makes a new draw. At the market price, demand is elastic at both the intensive and extensive margins upward, but only at the intensive margin downward: demand is kinked.

1.3 Equilibria

I now show that kinked demand curves lead to price indeterminacy. Consider the price-setting problem of a firm maximizing profits in the face of a kinked demand curve. I will rely on local optimality conditions. However, with no restriction on the distribution of search costs, it cannot in principles be ruled out that the profit function has multiple local maxima. To avoid this peripheral issue, I assume that the distribution of search costs is such that profits are single-peaked:

Assumption 3. *For any market price p^*, the profits function $\Pi^k(p^k) = (p^k - c)D^k(p^k, p^*)$ is single-peaked.*

A price p^* is an equilibrium price if, in response to all firms charging p^*, firm k finds it optimal to charge p^* too. Firm k sets its price to maximize profits $\Pi^k(p^k) = (p^k - c)D^k(p^k, p^*)$, which inherit the kink in demand at $p^k = p^*$. Because of this non-differentiability, the usual first-order condition is inapplicable, and the local optimality for k of charging p^* takes instead the weaker form that the left derivative be positive, and the right-derivative be negative:

$$\Pi^{k'}_-(p^*) = d(p^*) + d'(p^*)(p^* - c) \geq 0, \tag{6}$$

$$\Pi^{k'}_+(p^*) = d(p^*) + [d'(p^*) - f(0)d(p^*)^2](p^* - c) \leq 0, \tag{7}$$

where I used Roy's identity $v'(p^*) = -d(p^*)$ to write the right-derivative. Define the monopoly price p^M as the unique price maximizing the profits of a hypothetical monopolist facing the individual demand $d(p^k)$: $\Pi^M(p^k) = (p^k - c)d(p^k)$. It is characterized in the standard way as the price that cancels the derivative of monopolistic profits:

$$\Pi^{M'}(p^M) = d(p^M) + d'(p^M)(p^M - c) = 0. \tag{8}$$

Proposition 2. *Under assumptions (1)-(3), there is a continuum of market prices p^* consistent with equilibrium. All equilibrium prices are less than the monopoly price.*

Proof. The monopoly price p^M satisfies equation (6) (with equality), and any price $p^* < p^M$ satisfies it too (with a strict inequality). Any price $p^* > p^M$ is such that the reverse inequality holds so it cannot be an equilibrium price. Because the monopoly price p^M satisfies equation (7) with a strict inequality, by continuity there is an interval of prices around p^M that satisfy it too. □

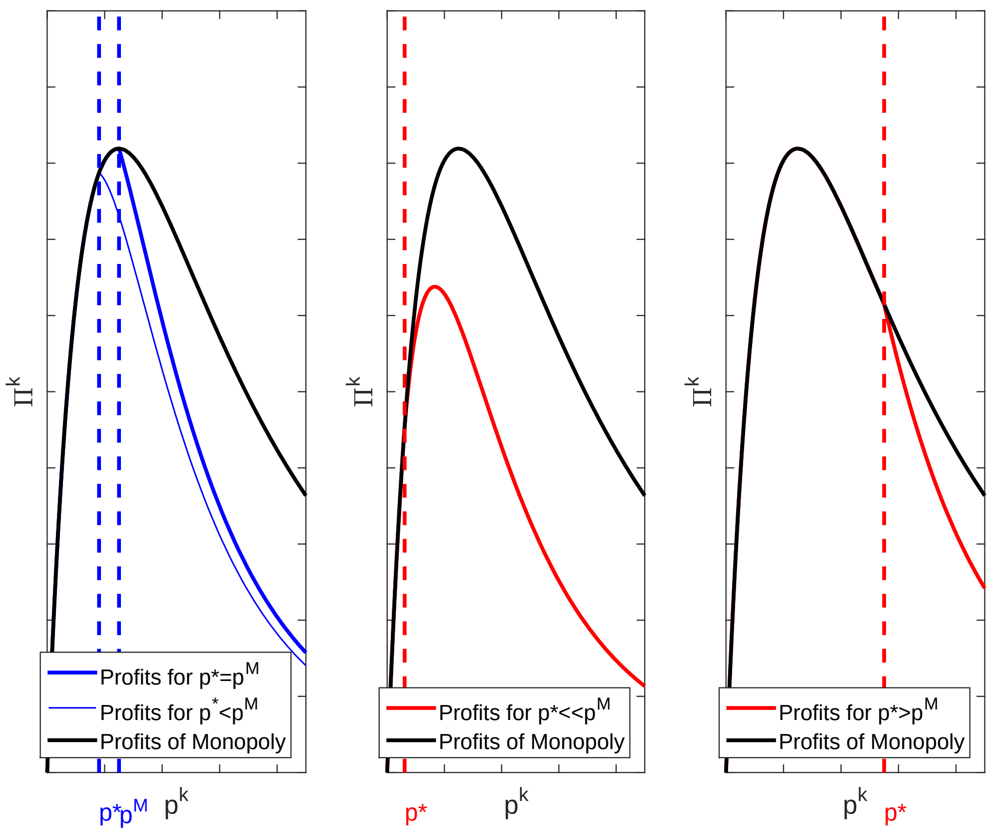

Figure 1: Profits function of firm k for different values of the market price p^*. The profits function is always kinked at p^*, but only for a range of values for p^* does firm k find it optimal to charge p^* in response.

A continuum of prices p^* are equilibrium prices because, in response to any of these prices, an individual firm has no incentives to charge a higher price—many of its customers would leave to competitors—nor to charge a lower price—it would fail to attract new customers. Yet not any price p^* is an equilibrium price. Figure 1 illustrates the logic behind the proof. The profits function of a firm k is always kinked at the market price p^*. However, only for an interval of values for p^* below the monopoly price p^M is the kink located at the top of the profits function and thus implies that firm k finds it optimal to charge p^* in response (left panel). For a sufficiently low market price p^*, firm k's best response is to charge a price in between p^* and p^M (middle panel). For a market price p^* above the monopoly price p^M, firm k's best response is to charge the monopoly price (right panel).

2 Discontinuous demand curves

I now take on a natural question: why don't earlier search models produce kinked-demand curves? Although Stiglitz (1979) and Woglom (1982) appeal to consumer search when they informally discuss the intuition for a kink, microfounded papers in the search literature did not find such a kink. As proposition 1 spells out, the kink in demand does require two weak assumptions on the distribution of costs, including continuously distributed search costs. I discuss the alternative assumption usually made in the consumer search literature—homogeneous search costs—starting with the seminal papers of Diamond (1971) and Burdett and Judd (1983).[3]

Proposition 1 uses assumption 1—search costs are continuously distributed—to guarantee that firms' demand curves are continuous. In contrast, the assumption that all consumers have the same search cost makes the distribution of search costs, and thus demand curves, discontinuous. Continuity is not the focus of this note—differentiability is—but looking at the case of discontinuously distributed search costs allows to explain what distinguishes the search model of this note from other search models.

Assume all consumers have the same search cost $\bar{\gamma}$—assume the distribution of search cost is a Dirac at $\bar{\gamma}$. In this case, all consumers who drew firm k buy at k if $v(p^k) \geq v(p^*) - \bar{\gamma}$, and draw another firm if $v(p^k) < v(p^*) - \bar{\gamma}$. Define the threshold price $p^T(p^*)$ (function of p^*) as the unique price satisfying $v(p^T) = v(p^*) - \bar{\gamma}$. (Necessarily, $p^T \geq p^*$.) Firm k's demand curve is:

$$D^k(p^k, p^*) = \begin{cases} d(p^k)dk & \text{if } p^k \leq p^T(p^*), \\ 0 & \text{if } p^k > p^T(p^*). \end{cases} \tag{9}$$

Firm k faces the monopolist's demand curve up to $p^k = p^T(p^*)$, at which point all its consumers at once judge the price sufficiently higher than the market price to make it worthwhile to search for another firm, and leave. Firm k's demand curve is not kinked, but instead discontinuous.

The discontinuity in demand leads, not to price indeterminacy, but to Diamond (1971)'s paradox: the unique equilibrium is for all firms to charge the monopoly price p^M. Indeed, although contrary to a monopoly, firm k loses all its customers if it increases its price by much (beyond $p^T(p^*)$), with positive search costs it loses none if it increases its price by less that what makes the benefits of search worth the cost (in between p^* and $p^T(p^*)$). In other words, firm k faces the monopolist's demand curve $d(p)$ locally around p^*. The key insight of Diamond's paradox is that, because only the marginal price increase matters to determine the equilibrium, it is enough for firm k to face the monopolist demand curve locally for it to price as a monopolist in equilibrium. Indeed, if all firms charge a price $p^* < p^M$, firm k's profits are strictly increasing at p^*, so firm k wants to increase its price and this is no equilibrium. And a price $p^* > p^M$ cannot be an equilibrium either: no firm wants to charge a price higher than the monopoly price.

To sum up, the standard assumption in search models of a common search cost creates a discontinuity in demand which hides the more generic property of a kink in demand when search costs are heterogeneous. The resulting Diamond's paradox hides the more generic indeterminacy result.

[3] The main result of Burdett and Judd (1983) is to depart from the framework of sequential search (which I restrict to here) in order to get price dispersion in equilibrium. Yet they do derive Diamond's paradox under sequential search as a benchmark, and maintain the assumption of a homogeneous search cost throughout their paper.

3 Ex-ante posted price vs. ex-post negotiated price

Recently, Hall (2005) and Shimer (2005) have rehabilitated the assumption of wage rigidity by showing that, once a labor search framework is purged of the ad hoc assumption of Nash-bargaining, a range of wages are consistent with equilibrium, making the assumption of wage rigidity robust to Barro (1977)'s critique that it is unreasonable to assume that employer-employee pairs would fail to realize mutually beneficial arrangements. The message is very close to the one of this paper: a search framework replaces the equilibrium price by a band of prices consistent with equilibrium. Yet, the source of the indeterminacy differs and for this reason does not apply to the same markets, as I now discuss.

3.1 The bilateral monopoly indeterminacy

In a labor market, the good is labor and the price is the wage, but obviously the difference between the indeterminacy of the kinked demand curve and the indeterminacy stressed by Hall and Shimer does not hinge on the change in vocabulary. Instead, in the search and matching framework that Hall and Shimer consider, the indeterminacy is driven by a key assumption: firms post vacancies but do not post wages with their job postings. Instead, the wage is assumed to be negotiated ex post after a firm and a worker have met. At the time of negotiation, the worker and the firm are in a situation of bilateral monopoly, and theory makes little restrictions on the wage that can come out of the negotiation process. It needs to be such that none of the two parties want to walk away from the deal, but the surplus of the match can be shared in an infinity of ways. Only by assuming an ad hoc bargaining rule—such as Nash bargaining—can the indeterminacy be resolved.

The kinked-demand indeterminacy is different from the bilateral-monopoly indeterminacy as it does not rely on the assumption that the terms of trade are bilaterally negotiated ex post. Instead, in the search model of this note, firms commit ex ante to a price which they publicly quote. Prices do arise from the interaction of multiple firms within a market, not from a private arrangement between two individuals. Yet competition between firms does not result in a unique price, because firms have difficulty competing prices down.

Whether a market is more prone to one or the other type of indeterminacy therefore depends on whether prices in this market are publicly quoted ex ante, or privately negotiated ex post. Broadly speaking, prices in goods markets are usually publicly quoted and thus prone to the kinked-demand indeterminacy, while wages in the labor market are usually negotiated privately, and thus prone to the bilateral monopoly indeterminacy.

3.2 From price indeterminacy to price rigidity

Both the bilateral monopoly indeterminacy and the kinked-demand indeterminacy open the door to price (or wage) rigidity. If a range of prices is consistent with equilibrium, then a given price is consistent with a range of values for costs. Therefore, there exist many equilibria where, in response to changes in costs, prices stay fixed, or move slowly. However, there also exist equilibria where prices are fully flexible. Price indeterminacy makes price rigidity possible, but not necessary. The prediction of price rigidity boils down to an issue of equilibrium selection. Why should we expect equilibria with some form of price rigidity to emerge from the price indeterminacy?

The kinked-demand indeterminacy offers a response to this question which is absent in the bilateral monopoly indeterminacy. With kinked-demand curves, a firm has a single best-response to a given market price—charging the market price—but many market prices are possible in equilibrium. The indeterminacy arises from strategic complementaries: firms have a strong incentive not to deviate from the price of competitors. In contrast, in the bilateral monopoly indeterminacy, the determination of the wage is precisely this: bilateral. It does not depend on the wages paid at other firms. There is no coordination problem.

The kinked-demand indeterminacy arises not only as a coordination problem, but a coordination problem of a particular form. When the past price is still an equilibrium price, a firm has an incentive to change its price only if the firm is confident that all other firms will do the same. It has no incentive to change its price if its price change is unilateral. Thus, keeping prices fixed is special among all equilibria: it is the equilibrium where a firm changes its price even if it alone to do so. It is the equilibrium to expect if firms are reluctant to do the first step in changing prices. In Dupraz (2016), I formalize this idea through an equilibrium selection criterion which I call adaptive rational expectations.

4 Conclusion

In this note, I have highlighted a property of markets with search frictions that adds to the microeonomic analysis of the price mechanism: consumer search leads not only to price dispersion but to price indeterminacy. Because firms cannot easily advertise price decreases, the competitive force that drives prices down in full-information models of competition is absent. Yet, prices do not necessarily rest at monopolistic levels: if all competitors charge a price below the monopoly price, an individual firm has often no incentives to charge a higher price, lest its customers would leave to competitors. The effective market price is the price firms end up coordinating on.

This matters for macroeconomic analysis too. Theories of the business cycle are often classified between classical equilibrium models and keynesian disequilibrium models, where disequilibrium most often refers to a temporary failure of prices to adjust to clear markets. Yet if there is no such thing as *the* equilibrium price, the distinction between equilibrium and disequilibrium becomes elusive. Shocks can move the economy within a continuum of equilibria, and, absent any new shock or policy intervention, no particular force drives the economy back to its initial point.

A Derivation of the reservation price policy

Note $V(p)$ the value for the consumer of holding a draw at price p. $V(p)$ is the maximum of the value of shopping at that firm, and of the (expected) value of drawing another firm:

$$V(p) = \max\{v(p), E(V) - \gamma\}. \tag{10}$$

The optimal search strategy is a threshold strategy: draw again whenever the price p is above a threshold $\bar{p}$ (when $v(p)$ is below a threshold $v(\bar{p})$). By definition, the threshold $\bar{p}$ satisfies:

$$v(\bar{p}) = E(V) - \gamma, \tag{11}$$

so that the value (10) of holding a draw at price p can be rewritten:

$$V(p) = \max\{v(p), v(\bar{p})\}. \tag{12}$$

Because the distribution of firms' prices is degenerated to a Dirac at p^*, this implies, taking expectations:

$$E(V) = \max\{v(p^*), v(\bar{p})\}. \tag{13}$$

Plugging this expression of $E(V)$ back into the definition of the threshold (11):

$$v(\bar{p}) = \max\{v(p^*), v(\bar{p})\} - \gamma. \tag{14}$$

This is an equation in $v(\bar{p})$ with unique solution:

$$v(\bar{p}) = v(p^*) - \gamma. \tag{15}$$

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